

Exercice 1 :

Calculer et simplifier si possible :

$$A = \left(-\frac{1}{2}\right) \times \left(-\frac{5}{7}\right) \quad B = \left(-\frac{2}{9}\right) \times \left(\frac{11}{5}\right) \quad C = \left(\frac{-4}{-7}\right) \times \left(-\frac{5}{9}\right) \quad D = -2 \times \left(-\frac{4}{5}\right)$$

$$E = 3 \times \frac{-5}{2} \times \frac{1}{-7} \quad F = -\frac{1}{2} \times \left(-\frac{5}{4}\right) \times \left(-\frac{7}{3}\right) \quad G = \left(-\frac{1}{2}\right) \times \left(-\frac{6}{9}\right) \quad H = \frac{4}{7} \times \left(-\frac{21}{8}\right)$$

Exercice 2 :

Calculer :

$$A = \frac{1}{9} - \frac{1}{6} \quad B = \frac{7}{12} + \frac{5}{18} \quad C = -\frac{20}{12} + \frac{2}{3} \quad D = \frac{5}{6} - \frac{-3}{-5}$$
$$E = -3 + \frac{7}{4} \quad F = -\frac{1}{8} + \frac{5}{12} \quad G = \frac{-5}{20} + \frac{9}{-6} \quad H = \frac{7}{9} - \frac{56}{12}$$

Exercice 3 :

Calculer $A = \frac{7}{3} + \frac{5}{6} - \frac{1}{8} + \frac{9}{4}$

Exercice 4 :

$$A = -\frac{3}{5} \times \left(-\frac{10}{9}\right) - \frac{1}{4} \times \left(1 - \frac{4}{5}\right) - \frac{3}{4} \quad B = -\frac{7}{12} + \frac{3}{4} \times \left(\frac{-2}{9}\right) \quad C = 4 \times \left(\frac{-21}{24}\right) - \frac{3}{2} \times \left(\frac{1}{4} + 8\right) - 1$$

Exercice 5 :

On pose $x = \frac{1}{2}$, $y = -\frac{1}{3}$, $z = -\frac{5}{4}$. Calculer :

a) $x + y + z$ b) $x - y - z$ c) $x - (y - z)$

Exercice 6 :

Quel est l'inverse de :

- a) -1 b) 1 c) 6 d) $\frac{5}{3}$

Exercice 7 :

$$A = \frac{\frac{3}{5}}{4} \quad B = \frac{\frac{3}{5}}{\frac{4}{5}} \quad C = \frac{\frac{11}{12}}{-\frac{55}{6}} \quad D = \frac{-\frac{7}{5}}{6 + \frac{1}{5}}$$

Exercice 8 :

- a) Soit MNP un triangle tel que $MN = \frac{1}{3}$, $MP = \frac{1}{4}$, $NP = \frac{1}{5}$. Le triangle MNP est-il rectangle ?
- b) Soit MNP un triangle tel que $MN = \frac{13}{20}$, $MP = \frac{3}{5}$, $NP = \frac{1}{4}$. Le triangle MNP est-il rectangle ?

Exercice 9 :

Soit n un nombre entier, $n > 0$.

Prouver que :
$$\frac{\frac{1}{n} - \frac{1}{n+1}}{\frac{1}{n+1} - \frac{1}{n+2}} = \frac{n+2}{n}.$$

Exercice 10 :

$$A = \left(\frac{3}{4} - \frac{2}{7} - 2 \right) \times \frac{21}{4} \quad B = -\frac{2}{3} + \frac{5}{3} \times \frac{12}{7} - 18 \times \frac{7}{27} \quad C = \frac{1 + \frac{2}{7} + \frac{3}{2}}{\frac{2}{3} + \frac{2}{21}}$$

Corrigé 1 :

$$\begin{array}{llll} A = \left(-\frac{1}{2}\right) \times \left(-\frac{5}{7}\right) & B = \left(-\frac{2}{9}\right) \times \left(\frac{11}{5}\right) & C = \left(\frac{-4}{-7}\right) \times \left(-\frac{5}{9}\right) & D = -2 \times \left(-\frac{4}{5}\right) & E = 3 \times \frac{-5}{2} \times \frac{1}{-7} \\ A = \frac{1}{2} \times \frac{5}{7} & B = -\frac{2}{9} \times \frac{11}{5} & C = \frac{4}{7} \times \left(-\frac{5}{9}\right) & D = 2 \times \frac{4}{5} & E = 3 \times \left(-\frac{5}{2}\right) \times \left(-\frac{1}{7}\right) \\ A = \frac{1 \times 5}{2 \times 7} & B = -\frac{2 \times 11}{9 \times 5} & C = -\frac{4}{7} \times \frac{5}{9} & D = \frac{2 \times 4}{5} & E = 3 \times \frac{5}{2} \times \frac{1}{7} \\ A = \frac{5}{14} & B = -\frac{22}{45} & C = -\frac{4 \times 5}{7 \times 9} & D = \frac{8}{5} & E = \frac{3 \times 5 \times 1}{2 \times 7} \\ & & C = -\frac{20}{63} & & E = \frac{15}{14} \end{array}$$

$$\begin{array}{lll} F = -\frac{1}{2} \times \left(-\frac{5}{4}\right) \times \left(-\frac{7}{3}\right) & G = \left(-\frac{1}{2}\right) \times \left(-\frac{6}{9}\right) & H = \frac{4}{7} \times \left(-\frac{21}{8}\right) \\ F = -\frac{1}{2} \times \frac{5}{4} \times \frac{7}{3} & G = \frac{1}{2} \times \frac{6}{9} & H = -\frac{4 \times 21}{7 \times 8} \\ F = -\frac{1 \times 5 \times 7}{2 \times 4 \times 3} & G = \frac{1 \times 6}{2 \times 9} & H = -\frac{4 \times 3 \times 7}{7 \times 2 \times 4} \\ F = -\frac{35}{24} & G = \frac{1 \times 2 \times 3}{2 \times 3 \times 3} & H = -\frac{3}{2} \\ & G = \frac{1}{3} & \end{array}$$

Corrigé 2 :

$$\begin{array}{llll} A = \frac{1}{9} - \frac{1}{6} & B = \frac{7}{12} + \frac{5}{18} & C = -\frac{20}{12} + \frac{2}{3} & D = \frac{5}{6} - \frac{-3}{-5} \\ A = \frac{1 \times 2}{9 \times 2} - \frac{1 \times 3}{6 \times 3} & B = \frac{7 \times 3}{12 \times 3} + \frac{5 \times 2}{18 \times 2} & C = -\frac{20}{12} + \frac{2 \times 4}{3 \times 4} & D = \frac{5}{6} - \frac{3}{5} \\ A = \frac{2}{18} - \frac{3}{18} & B = \frac{21}{36} + \frac{10}{36} & C = -\frac{20}{12} + \frac{8}{12} & D = \frac{5 \times 5}{6 \times 5} - \frac{3 \times 6}{5 \times 6} \\ A = \frac{2-3}{18} & B = \frac{21+10}{36} & C = \frac{-20+8}{12} & D = \frac{25-18}{30} \\ A = -\frac{1}{18} & B = \frac{31}{36} & C = -\frac{12}{12} & D = \frac{25}{30} - \frac{18}{30} \\ & & C = -1 & D = \frac{7}{30} \end{array}$$

$$E = -3 + \frac{7}{4}$$

$$E = \frac{-3 \times 4}{4 \times 1} + \frac{7}{4}$$

$$E = \frac{-12}{4} + \frac{7}{4}$$

$$E = \frac{-12 + 7}{4}$$

$$E = -\frac{5}{4}$$

$$F = -\frac{1}{8} + \frac{5}{12}$$

$$F = -\frac{1 \times 3}{8 \times 3} + \frac{5 \times 2}{12 \times 2}$$

$$F = -\frac{3}{24} + \frac{10}{24}$$

$$F = \frac{10 - 3}{24}$$

$$F = \frac{7}{24}$$

$$G = \frac{-5}{20} + \frac{9}{-6}$$

$$G = \frac{-5}{20} - \frac{9}{6}$$

$$G = \frac{-5 \times 3}{20 \times 3} - \frac{9 \times 10}{6 \times 10}$$

$$G = \frac{-15}{60} - \frac{90}{60}$$

$$G = \frac{-15 - 90}{60}$$

$$G = -\frac{105}{60}$$

$$G = -\frac{15 \times 7}{15 \times 4} = -\frac{7}{4}$$

$$H = \frac{7}{9} - \frac{56}{12}$$

$$H = \frac{7 \times 4}{9 \times 4} - \frac{56 \times 3}{12 \times 3}$$

$$H = \frac{28}{36} - \frac{168}{36}$$

$$H = \frac{28 - 168}{36}$$

$$H = -\frac{140}{36}$$

$$H = -\frac{35}{9}$$

Corrigé 3 :

$$A = \frac{7}{3} + \frac{5}{6} - \frac{1}{8} + \frac{9}{4}$$

$$A = \frac{7 \times 2}{3 \times 2} + \frac{5}{6} - \frac{1}{8} + \frac{9 \times 2}{4 \times 2}$$

$$A = \frac{14}{6} + \frac{5}{6} - \frac{1}{8} + \frac{18}{8}$$

$$A = \frac{14 + 5}{6} + \frac{18 - 1}{8}$$

$$A = \frac{19}{6} + \frac{17}{8}$$

$$A = \frac{19 \times 4}{6 \times 4} + \frac{17 \times 3}{8 \times 3}$$

$$A = \frac{76}{24} + \frac{51}{24}$$

$$A = \frac{127}{24}$$

Corrigé 4 :

$$A = -\frac{3}{5} \times \left(-\frac{10}{9}\right) - \frac{1}{4} \times \left(1 - \frac{4}{5}\right) - \frac{3}{4}$$

$$A = \frac{3 \times 10}{5 \times 9} - \frac{1}{4} \times \left(\frac{5}{5} - \frac{4}{5}\right) - \frac{3}{4}$$

$$A = \frac{3 \times 5 \times 2}{5 \times 3 \times 3} - \frac{1}{4} \times \left(\frac{5-4}{5}\right) - \frac{3}{4}$$

$$A = \frac{2}{3} - \frac{1}{4} \times \frac{1}{5} - \frac{3}{4}$$

$$A = \frac{2}{3} - \frac{1 \times 1}{4 \times 5} - \frac{3}{4}$$

$$A = \frac{2}{3} - \frac{1}{20} - \frac{3}{4}$$

$$A = \frac{2 \times 20}{3 \times 20} - \frac{1 \times 3}{20 \times 3} - \frac{3 \times 15}{4 \times 15}$$

$$A = \frac{40}{60} - \frac{3}{60} - \frac{45}{60}$$

$$A = \frac{40 - 3 - 45}{60}$$

$$A = \frac{-8}{60} = \frac{-4 \times 2}{4 \times 15} = \frac{-2}{15}$$

$$B = -\frac{7}{12} + \frac{3}{4} \times \left(\frac{-2}{9}\right)$$

$$B = -\frac{7}{12} + \frac{3}{4} \times \left(-\frac{2}{9}\right)$$

$$B = -\frac{7}{12} - \frac{3 \times 2}{4 \times 9}$$

$$B = -\frac{7}{12} - \frac{3 \times 2}{2 \times 2 \times 3 \times 3}$$

$$B = -\frac{7}{12} - \frac{1}{6}$$

$$B = -\frac{7}{12} - \frac{1 \times 2}{6 \times 2}$$

$$B = -\frac{7}{12} - \frac{2}{12}$$

$$B = \frac{-7-2}{12}$$

$$B = \frac{-9}{12} = -\frac{3 \times 3}{4 \times 3} = -\frac{3}{4}$$

$$C = 4 \times \left(\frac{-21}{24}\right) - \frac{3}{2} \times \left(\frac{1}{4} + 8\right) - 1$$

$$C = -\frac{4 \times 21}{24} - \frac{3}{2} \times \left(\frac{1}{4} + \frac{8 \times 4}{1 \times 4}\right) - 1$$

$$C = -\frac{4 \times 3 \times 7}{3 \times 2 \times 4} - \frac{3}{2} \times \left(\frac{1+32}{4}\right) - 1$$

$$C = -\frac{7}{2} - \frac{3}{2} \times \frac{33}{4} - 1$$

$$C = -\frac{7}{2} - \frac{3 \times 33}{2 \times 4} - 1$$

$$C = -\frac{7}{2} - \frac{99}{8} - 1$$

$$C = -\frac{7 \times 4}{2 \times 4} - \frac{99}{8} - \frac{8}{8}$$

$$C = -\frac{28}{8} - \frac{99}{8} - \frac{8}{8}$$

$$C = \frac{-28-99-8}{8}$$

$$C = \frac{-135}{8}$$

Corrigé 5 :

$$A = x + y + z$$

$$A = \frac{1}{2} + \left(-\frac{1}{3}\right) + \left(-\frac{5}{4}\right)$$

$$A = \frac{1}{2} - \frac{1}{3} - \frac{5}{4}$$

$$A = \frac{1 \times 6}{2 \times 6} - \frac{1 \times 4}{3 \times 4} - \frac{5 \times 3}{4 \times 3}$$

$$A = \frac{6}{12} - \frac{4}{12} - \frac{15}{12}$$

$$A = \frac{6 - 4 - 15}{12}$$

$$A = \frac{-13}{12} = -\frac{13}{12}$$

$$B = x - y - z$$

$$B = \frac{1}{2} - \left(-\frac{1}{3}\right) - \left(-\frac{5}{4}\right)$$

$$B = \frac{1}{2} + \frac{1}{3} + \frac{5}{4}$$

$$B = \frac{1 \times 6}{2 \times 6} + \frac{1 \times 4}{3 \times 4} + \frac{5 \times 3}{4 \times 3}$$

$$B = \frac{6}{12} + \frac{4}{12} + \frac{15}{12}$$

$$B = \frac{6 + 4 + 15}{12}$$

$$B = \frac{25}{12}$$

$$C = x - (y - z)$$

$$C = \frac{1}{2} - \left[-\frac{1}{3} - \left(-\frac{5}{4}\right)\right]$$

$$C = \frac{1}{2} - \left(-\frac{1}{3} + \frac{5}{4}\right)$$

$$C = \frac{1}{2} - \left(-\frac{1 \times 4}{3 \times 4} + \frac{5 \times 3}{4 \times 3}\right)$$

$$C = \frac{1}{2} - \left(-\frac{4}{12} + \frac{15}{12}\right)$$

$$C = \frac{1}{2} - \left(\frac{15 - 4}{12}\right)$$

$$C = \frac{1}{2} - \frac{11}{12}$$

$$C = \frac{1 \times 6}{2 \times 6} - \frac{11}{12}$$

$$C = \frac{6}{12} - \frac{11}{12}$$

$$C = \frac{6 - 11}{12}$$

$$C = -\frac{5}{12}$$

Corrigé 6 :

L'inverse de -1 est $\frac{1}{-1} = -\frac{1}{1} = -1$

L'inverse de 1 est $\frac{1}{1} = 1$

L'inverse de 6 est $\frac{1}{6}$

L'inverse de $\frac{5}{3}$ est $\frac{3}{5}$.

Corrigé 7 :

$$A = \frac{\frac{3}{5}}{4} = \frac{3}{5} \times \frac{1}{4} = \frac{3 \times 1}{5 \times 4} = \frac{3}{20}$$

$$B = \frac{\frac{3}{5}}{\frac{4}{5}} = 3 \times \frac{4}{5} = \frac{3 \times 4}{5} = \frac{12}{5}$$

$$C = \frac{\frac{11}{12}}{-\frac{55}{6}} = \frac{11}{12} \times \left(-\frac{6}{55} \right) = -\frac{11 \times 6}{12 \times 55} = -\frac{11 \times 6}{2 \times 6 \times 11 \times 5} = -\frac{1}{10}$$

$$D = \frac{-\frac{7}{5}}{6 + \frac{1}{5}} = \frac{-\frac{7}{5}}{\frac{6 \times 5}{1 \times 5} + \frac{1}{5}} = \frac{-\frac{7}{5}}{\frac{30}{5} + \frac{1}{5}} = \frac{-\frac{7}{5}}{\frac{31}{5}} = -\frac{7}{5} \times \frac{5}{31} = -\frac{7 \times 5}{5 \times 31} = -\frac{7}{31}$$

Corrigé 8 :

1) Dans le triangle MNP, le côté [MN] est le plus long. On a $MN^2 = \left(\frac{1}{3} \right)^2 = \frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$

$$\text{De plus, } MP^2 + NP^2 = \left(\frac{1}{4} \right)^2 + \left(\frac{1}{5} \right)^2 = \frac{1}{4} \times \frac{1}{4} + \frac{1}{5} \times \frac{1}{5} = \frac{1}{16} + \frac{1}{25} = \frac{1 \times 25}{16 \times 25} + \frac{1 \times 16}{25 \times 16} = \frac{25}{400} + \frac{16}{400} = \frac{41}{400}$$

$$\text{A-t-on } \frac{1}{9} = \frac{41}{400} ?$$

$$\text{On a : } \begin{array}{l} 1 \times 400 = 400 \\ 9 \times 41 = 369 \end{array}$$

On constate que $1 \times 400 \neq 9 \times 41$ donc les fractions $\frac{1}{9}$ et $\frac{41}{400}$ ne sont pas égales.

Ainsi, $MN^2 \neq MP^2 + NP^2$

Le triangle MNP n'est donc pas rectangle.

2) Dans le triangle MNP, le côté [MN] est le plus long. On a $MN^2 = \left(\frac{13}{20}\right)^2 = \frac{13}{20} \times \frac{13}{20} = \frac{169}{400}$

De plus, $MP^2 + NP^2 = \left(\frac{3}{5}\right)^2 + \left(\frac{1}{4}\right)^2 = \frac{3}{5} \times \frac{3}{5} + \frac{1}{4} \times \frac{1}{4} = \frac{9}{25} + \frac{1}{16} = \frac{9 \times 16}{25 \times 16} + \frac{1 \times 25}{16 \times 25} = \frac{144}{400} + \frac{25}{400} = \frac{169}{400}$

Ainsi, $MN^2 = MP^2 + NP^2$

D'après la réciproque du théorème de Pythagore, le triangle MNP est rectangle en P.

Corrigé 9 :

$$\text{On a } \frac{1}{n} - \frac{1}{n+1} = \frac{1 \times (n+1)}{n \times (n+1)} - \frac{1 \times n}{n \times (n+1)} = \frac{n+1}{n(n+1)} - \frac{n}{n(n+1)} = \frac{n+1-n}{n(n+1)} = \frac{1}{n(n+1)}$$

$$\frac{1}{n+1} - \frac{1}{n+2} = \frac{1 \times (n+2)}{(n+1) \times (n+2)} - \frac{1 \times (n+1)}{(n+1) \times (n+2)} = \frac{n+2}{(n+1)(n+2)} - \frac{n+1}{(n+1)(n+2)} = \frac{n+2-(n+1)}{(n+1)(n+2)} = \frac{n+2-n-1}{(n+1)(n+2)}$$

$$\text{Donc } \frac{1}{n+1} - \frac{1}{n+2} = \frac{1}{(n+1)(n+2)}$$

$$\text{Ainsi, } \frac{\frac{1}{n} - \frac{1}{n+1}}{\frac{1}{n+1} - \frac{1}{n+2}} = \frac{\frac{1}{n(n+1)}}{\frac{1}{(n+1)(n+2)}} = \frac{1}{n(n+1)} \times \frac{(n+1)(n+2)}{1} = \frac{(n+1)(n+2)}{n(n+1)} = \frac{n+2}{n}$$

Corrigé 10 :

$$A = \left(\frac{3}{4} - \frac{2}{7} - 2 \right) \times \frac{21}{4}$$

$$A = \left(\frac{3 \times 7}{4 \times 7} - \frac{2 \times 4}{7 \times 4} - \frac{2 \times 28}{1 \times 28} \right) \times \frac{21}{4}$$

$$A = \left(\frac{21}{28} - \frac{8}{28} - \frac{56}{28} \right) \times \frac{21}{4}$$

$$A = \left(\frac{21 - 8 - 56}{28} \right) \times \frac{21}{4}$$

$$A = -\frac{43}{28} \times \frac{21}{4}$$

$$A = -\frac{43 \times 3 \times 7}{4 \times 7 \times 4}$$

$$A = -\frac{129}{16}$$

$$B = -\frac{2}{3} + \frac{5}{3} \times \frac{12}{7} - 18 \times \frac{7}{27}$$

$$B = -\frac{2}{3} + \frac{5 \times 12}{3 \times 7} - \frac{18 \times 7}{27}$$

$$B = -\frac{2}{3} + \frac{5 \times 4 \times 3}{3 \times 7} - \frac{9 \times 2 \times 7}{9 \times 3}$$

$$B = -\frac{2}{3} + \frac{20}{7} - \frac{14}{3}$$

$$B = \frac{-2 - 14}{3} + \frac{20}{7}$$

$$B = \frac{-16}{3} + \frac{20}{7}$$

$$B = \frac{-16 \times 7}{3 \times 7} + \frac{20 \times 3}{7 \times 3}$$

$$B = \frac{-112}{21} + \frac{60}{21}$$

$$B = \frac{-112 + 60}{21}$$

$$B = \frac{-52}{21} = -\frac{52}{21}$$

$$\text{On a : } 1 + \frac{2}{7} + \frac{3}{2} = \frac{7}{7} + \frac{2}{7} + \frac{3}{2} = \frac{7+2}{7} + \frac{3}{2} = \frac{9}{7} + \frac{3}{2} = \frac{9 \times 2}{7 \times 2} + \frac{3 \times 7}{2 \times 7} = \frac{18+21}{14} = \frac{39}{14}$$

$$\text{D'autre part, } \frac{2}{3} + \frac{2}{21} = \frac{2 \times 7}{3 \times 7} + \frac{2}{21} = \frac{14}{21} + \frac{2}{21} = \frac{16}{21}$$

$$\text{On en déduit que } C = \frac{1 + \frac{2}{7} + \frac{3}{2}}{\frac{2}{3} + \frac{2}{21}} = \frac{\frac{39}{14}}{\frac{16}{21}} = \frac{39}{14} \times \frac{21}{16} = \frac{39 \times 21}{14 \times 16} = \frac{39 \times 3 \times 7}{7 \times 2 \times 16} = \frac{117}{32}$$